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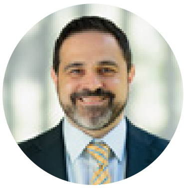
Combining active managers: A practical approach

- This research paper bridges a critical gap in the existing literature related to combining active managers in an investor's portfolio. Although extensive research has been conducted on asset allocation strategies and how to pick active managers, there is a very limited body of literature regarding the process of combining active managers within a portfolio. We have identified existing asset allocation techniques that can be used for systematic active bundling.
- Our framework begins by emphasizing the significance of key statistics for active managers, which provide a foundational understanding of how managers can be combined. Subsequently, the paper explores a variety of techniques and their relevant parameters.
- Finally, we employ the Vanguard Asset Allocation Model (VAAM), a proprietary model for determining asset allocation among active, passive, and factor investments, to illustrate how different bundling methods can lead to distinctly varied portfolio allocations. Although the VAAM is not used as a method to optimally allocate among active managers, the findings from this stage of the research offer a valuable perspective to help investors understand the impact that active bundling has on strategic asset allocation.

Authors

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Background

In a world of constantly changing and evolving investment opportunities, both individuals and institutional investors seek to construct well-diversified and high-performing portfolios. Passive strategic asset allocation forms the foundation of portfolio construction, with research showing that over 80% of variation in return is explained by the policy portfolio.[1] However, many people believe that variability in return is just a part of the asset allocation story. Ibbotson and Kaplan (2000) extended their study and found that only about 40% of the return variation between funds is due to asset allocation, with the rest of the cumulative return balance due to other factors, including style, timing, and security selection.

As a result, the optimal portfolio solution for many investors does not end with the passive strategic portfolio building blocks, so they venture into active asset allocation and selection. While strategic asset allocation establishes the foundation, it is the subsequent inclusion of active management that presents an opportunity to further enhance the performance of a portfolio, galvanized by generating "alpha," or a positive excess return.[2]

This paper assumes that an investor has already identified the "winning" managers that are expected to outperform their prospective benchmarks. Accordingly, it tackles the question of how to combine active managers with the rest of the portfolio. We first delve into factors that are important in selecting the technique for combining active managers, and then provide an overview of the different approaches that can be used to combine active strategies. Furthermore, we test-run the selection of active funds with each approach and show the resulting active portfolio. Lastly, we combine the active bundle with the passive allocation to form a holistic portfolio, in order to show how different active bundling techniques impact the active/passive mix.

Before diving into specific methods, let's address the fundamental reasons why it matters to combine active managers effectively. While extensive practitioner research is available on selecting individual managers, it often falls short in regard to combining these managers in a portfolio. Investors are left relying on heuristic and imprecise rules. Our goal is to establish a framework that empowers investors to combine managers with comprehensive and consistent approaches.

Key factors that influence active bundling

Since there is a considerable body of literature related to picking active managers, we do not cover that topic in this paper. Therefore, to provide practical portfolio construction approaches for combining multiple active managers, we assume that the best managers have already been identified. These days, most investor practices are based on building active multimanager bundles grounded in fundamentals and a few rules of thumb. The limited literature is often very theoretical and struggles to provide a practical way of combining active managers. Thus, the question we intend to answer is how an investor should optimally blend a given set of active managers for a given asset or sub-asset class. Although examples throughout this paper are focused on equities, the techniques outlined are applicable to all asset and sub-asset classes.

A critical starting point in assessing active bundling techniques is to take a deep dive into individual managers to understand their approach and exposures. A seminal study by Fama and French (1993) serves as a valuable starting point to systematically evaluate managers. Based on their work, active manager performance can be broken down into the three components (alpha, beta, and factor exposures) described in the following formula:

$$R_i = \alpha_i + \beta_i R_M + \sum_{f=1}^{F} \theta_{i,f}\,\varphi_{i,f} + \varepsilon_i$$

1 See Brinson, Hood, and Beebower (1986); and Ibbotson and Kaplan (2000).

2 Excess return includes active factor and alpha returns relative to the policy portfolio or benchmark.

In this equation, R_i is the return of the active fund i, R_M is the return of the market portfolio and β_i is the active manager's sensitivity to it, α_i is the active manager's skill (which can be positive or negative), $\varphi_{i,f}$ represents the excess return component to factor f, and $\theta_{i,f}$ is the active funds' sensitivity to it (i.e., factor loading or exposure). Finally, ε_i represents the additional component of idiosyncratic active risk and is centered at zero.

Breaking down the manager's return into alpha, market exposure (also known as beta), and factors creates a systematic way to look at performance from an exposure and consistency standpoint. This knowledge not only provides a good foundation for understanding what managers do, but also helps with evaluating the relationship between different managers that are considered for an active bundle. To make exposures come to life, we have selected a list of anonymized U.S. active equity funds as examples. In **Figure 1**, we present a snapshot of these funds' exposures and some related statistics.

FIGURE 1

Morningstar style-box factor weight and risk characteristics for anonymized U.S. active equity funds

	Morningstar style-box factor weight				Risk characteristics				
	Size	Value	Blend	Growth	Market exposure (Beta)	Factor risk (%)	Specific risk (%)	Total risk (%)	Active risk (%)
Large Value	Large	35%	32%	3%	0.84	15.10	2.22	15.26	6.63
	Mid	15%	11%	1%					
	Small	2%	1%	0%					
Large Blend	Large	14%	25%	32%	0.91	15.50	4.00	16.01	4.99
	Mid	5%	20%	0%					
	Small	2%	1%	0%					
Large Growth	Large	9%	17%	34%	0.95	16.11	4.72	16.79	5.38
	Mid	3%	22%	9%					
	Small	2%	1%	1%					
Small Blend	Large	0%	0%	0%	1.20	21.98	3.59	22.27	10.00
	Mid	8%	10%	2%					
	Small	23%	46%	11%					
Small Growth	Large	0%	0%	1%	1.25	22.07	3.21	22.31	8.60
	Mid	3%	9%	18%					
	Small	11%	32%	26%					
Total U.S. Market (passive index)	Large	13%	21%	38%	1.00	16.71	1.68	16.79	0.00
	Mid	6%	9%	5%					
	Small	3%	3%	2%					

Notes: Beta and active risk are relative to the CRSP US Total Market Index as of December 31, 2023. Specific risk, or idiosyncratic risk, is the residual risk related to the asset. The larger-than-expected specific risk for the total market is attributed to the universe- and risk-estimation method.

Sources: Vanguard calculations, based on data from Morningstar and MSCI Barra.

Before delving into the realm of optimization techniques for active managers, we want to establish a solid understanding of certain fundamental concepts related to a fund's exposure and risk components. In fact, understanding the components of Figure 1 can carry an investor far in their portfolio construction journey:

- **Market capitalization (Morningstar style box)** is one of the most recognizable classification systems in finance. Its importance is traced to Fama and French (1993), who highlighted how market capitalization (company size) and valuation are key variables to explain the performance of active managers. Knowing style-box composition can be helpful when trying to identify how funds can be different from or complementary to each other. For example, while classification can state that a fund belongs in the Small-Blend category (see Figure 1), one can observe that not all assets are categorized in that "box." In fact, about 20% of them are classified as mid-cap.
- **Market exposure (Beta)** is a useful measure when constructing investment portfolios, helping investors understand how a security, manager, or portfolio may behave compared with the broader market. All else being equal, a beta less than 1.0 implies that a portfolio is expected to be less volatile than a benchmark, while a beta over 1.0 indicates that it should be more volatile. This systematic measure is crucial when making portfolio construction decisions and helps explain a significant amount of an active manager's risk.
- **Factor and specific risks** are essential components used to assess and manage the overall risk and exposures of an investment bundle. Like a Morningstar style box, factor risk represents the exposure of the manager to broad market factors, but it goes beyond size and valuations. Factors can include a range of macro and asset-specific exposures that can affect your portfolio. Knowing these factors can help an investor make a well-informed decision regarding what factors they want exposure to in the long-term. On the other hand, specific risk, also called unsystematic or idiosyncratic risk, refers to the risk that cannot be explained by factors. This risk is borne by individual assets within a portfolio. Unlike factor risk, it can be reduced and even eliminated through diversification. Notably, active managers can deliver alpha through both factor and specific risks, but knowing the breakdown in advance can be useful when creating an active bundle or building a holistic portfolio.
- **Active risk (also known as excess volatility or tracking error)** serves as a crucial tool for investors and portfolio managers to assess the performance and risk of an investment portfolio relative to its benchmark. Low active risk implies that the portfolio closely matches benchmark performance, while high active risk can be an indicator of significant divergence. The magnitude of active risk can be linked to risk measures previously mentioned, such as market exposure, factors, and specific risk.

While we will discuss a variety of approaches in this paper, the above statistics are key to understanding why different methods produce certain allocations within the active bundles. The methodology behind all active bundles needs to be transparent and easy to explain. Therefore, it is highly dependent on assumptions and confidence in those assumptions. Most of the above variables are grounded in risk or categorization. For more robust models and solutions, investors need to accurately predict total or active return. However, extensive literature points to the difficulties of estimating returns with any degree of precision. For this reason, methods that do not rely on investors' views on assets' future returns have become popular.

Bundling techniques

In this paper, we will examine a range of bundling approaches that cover a variety of goals and degrees of data conviction. **Figure 2** summarizes techniques, inputs, and considerations for each strategy. This is not an exhaustive list, but we believe that it offers a good starting point for most cases that investors may want to explore. With each of these approaches, except the input-free (or "1/N") approach, it is important to note that the inputs are time varying and must be reevaluated at a predetermined frequency.

FIGURE 2

Comparing optimization techniques based on inputs and ease of implementation

Optimization technique	Ease of implementation	Inputs: Return	Inputs: Volatility	Inputs: Correlations	Inputs: Considerations/ dependencies
Input-free (1/N) approach	**Simple**	—	—	—	—
Market capitalization	↓	—	—	—	• Map into style box
Inverse volatility	↓	—	✓	—	• Volatility estimation
Minimum tracking error	↓	—	✓	✓	• Benchmark • Risk estimation • Optimization
Risk parity	↓	—	✓	✓	• Benchmark (if relative) • Risk estimation • Optimization
Outcome risk parity	↓	✓	✓	✓	• Benchmark • Risk estimation • Optimization • Total return • Return autocorrelation
Mean-variance	↓	✓	✓	✓	• Benchmark (if relative) • Risk estimation • Optimization • Risk aversion • Alpha or total return
Maximum information or Sharpe ratio	**Complex**	✓	✓	✓	• Benchmark (if relative) • Risk estimation • Optimization • Alpha or total return

Note: See the Appendix for a summary of pros and cons related to each technique.
Source: Vanguard.

Blending active managers: From naive to trusting the "Greeks"

In this section, we review the bundling approaches illustrated in Figure 2, which can be used to allocate across different active strategies or managers. Most of these approaches are methods that academic research applies to single securities and asset class allocation, while others are more specific to allocation by active managers.

We begin with the simplest approaches, which can be a powerful starting point, and then progress to those that are more complex.

Simple: The 1/N approach

The 1/N approach is the simplest that we cover in this paper, and states that each active manager should be weighted equally in the bundle. For example, if an investor has to allocate wealth across seven different U.S. equity active managers, they will simply weight each of them roughly 14.3% (one-seventh) of the invested amount. The rationale for this approach is straightforward: It requires minimum effort and allows for some diversification. Generally, a larger number ("N") should lead to greater diversification.

This method is often applied to single securities and became popular in the late 2000s when DeMiguel, Garlappi, and Uppal (2009) found that naive 1/N strategies outperformed optimized solutions for U.S. stock selection. Over the years, their findings have been criticized on multiple occasions, and several studies have even concluded the opposite.[3] Academics and practitioners continue to debate whether out-of-sample, optimized solutions actually outperform the naive, input-free approaches. We believe that in the absence of any information or with very low confidence in the available information, this approach is a reasonable starting point.

Market capitalization: Morningstar style-box-inspired approach

The study by Fama and French (1993) introduced the three-factor model, which explains key drivers of cross-sectional returns through sensitivity to the market, size, and valuations. This finding represented the origin for market-capitalization-inspired portfolio construction, where the investment allocation is conducted across segments such as geographic region, valuations, and market capitalization. This approach requires knowledge of the underlying characteristics of the portfolio's investments, such as allocation within the Morningstar style box. Based on this knowledge, an investor can combine assets to align with the exposures of a target benchmark, with the initial goal to be "market-cap" neutral for two factors: size and valuations.

There are many reasons why market-capitalization-inspired portfolio construction is popular. It is a simple way to construct portfolios, while being cost effective due to the limited need for rebalancing, and can lead to a fairly diversified allocation. However, drawbacks to relying solely on market capitalization include concentration risk and a lack of consideration for alpha.

In Figure 1, we displayed the Morningstar style-box composition for various active funds alongside the breakdown of the total market. Leveraging this data and using the minimization technique depicted in the formula below, investors can systematically allocate assets among active managers, aiming to closely align with the total market capitalization.

$$\min_{\{x\}} \sum_{i=1}^{n} \sum_{s=1}^{9} \left(x_i F_{i,s} - y_s \right)^2$$

In this equation, n is the number of active managers, x_i is the weight of active manager i, $F_{i,s}$ is the active manager's Morningstar style-box factor s weight, and y_s is the benchmark's factor s weight.

3 See Kritzman, Page, and Turkington (2010); and Fugazza, Guidolin, and Nicodano (2015).

An important aspect of this approach is that while we use the example based on market capitalization, the same approach could be applied to a variety of factor exposures, given the availability of a risk model.

This approach is easy to implement and can be applied systematically. But depending on the available active managers, it does not necessarily lead to a unique solution and can exhibit significant concentration in one of the selected active managers. As a result, additional ad hoc rules might be necessary to arrive at a reasonable allocation within the active bundle.

Volatility matters: Inverse-volatility approach

The inverse-volatility approach incorporates volatility expectations as an input and is similar to the 1/N approach, except that each active manager's weight is inversely proportional to the level of the manager's total risk (see the formula below).

$$x_i = \frac{\frac{1}{\sigma_i}}{\sum_{i=1}^{n} \frac{1}{\sigma_i}}$$

In this equation, σ is the total volatility for each active strategy.

With this approach, the lowest weight is given to active funds with the most volatility, while low-volatility active funds will compose a much larger portion of the portfolio. It can generally be viewed as an attempt to achieve a portfolio with more diversified and balanced risk exposures.

The inverse-volatility approach could also be applied to the volatility of the active return, instead of the total return volatility of the active strategy. Using the volatility of the excess return would lead to a different allocation compared with using the total return volatility. Potential downsides of the inverse-volatility approach include that it will overemphasize managers with lower volatility or tracking error, and not account for aspects such as correlations among active strategies or the potential to generate higher alpha.

When volatility and correlation matter: Risk-parity approach

Standard risk-parity approaches aim to equalize the contribution to total portfolio risk from each asset (Qian, 2005, 2011). Risk is normally defined as the volatility of returns, although other risk measures have also been used (e.g., tail risk). In the active funds allocation space, building a risk-parity portfolio first requires deconstructing the portfolio risk into amounts attributable to each active fund. $AR(x_i)$ is some measure of risk as a function of the weights denoted, and AMC_i (x_i) is the marginal risk contribution of active fund i. The marginal risk contribution is then multiplied by the asset weight to define the risk contribution, $ARC_i\ (x_i)$, for each asset as described in the formula below.

At this point, we can solve for the set of weights that equalizes these contributions. Similar to the approaches reported above, we apply additional constraints such that the portfolio is long only, and the weights sum to 1.

$$x_i = \begin{cases} \frac{1}{n}\ AR(x_i) = ARC_i(x_i),\ \forall i \in \{1,\ldots,n\} \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases}$$

For our purposes, we can define the problem in multiple ways, which can be applied to total exposures, total active exposures, and just in the excess-return space. Risk-parity approaches can then be applied in a classical form such that $AR(x)$ is equal to the portfolio total level of volatility or, alternatively, such that $AR(x)$ is equal to the portfolio tracking error (TE_p).

Risk parity with return predictability: Outcome risk-parity approach

As previously mentioned, the broadest definition of risk parity is a portfolio construction approach in which each asset contributes equally to some risk measure. The most well-known risk measure is portfolio volatility, but this comes with an important caveat: It may not be a suitable measure of risk when active managers' returns demonstrate autocorrelation over multiple periods.

For example, consider three active investment opportunities with differing levels of autocorrelation. One asset's returns follow momentum, another's follow mean reversion, and the third's demonstrate no autocorrelation. Even if the assets have identical expected returns and volatilities, the distribution of possible returns or wealth outcomes will be narrower for the mean-reverting opportunity and wider for the momentum opportunity. Therefore, investors will likely prefer active strategies that feature the mean-reverting-returns asset, because these strategies offer narrower outcomes but the same level of expected returns.

To account for this, Renzi-Ricci, Harvey, and Baynes (2024) introduced a new approach termed "outcome risk parity." Outcome risk parity measures risk by the dispersion of return outcomes, which captures the additional uncertainty due to return autocorrelation.

The optimization problem is then the same as a traditional risk-parity approach but where $AR(x_i)$, which is the measure of active risk as a function of the weights, is now defined as in the following formula:

$$AR(x_i) = Stdev\left(\left\{\sum_{i=1}^{n} x_i R_{s,i}\right\}_{s \in \{1,\ldots,S\}}\right)$$

In this equation, $Stdev(\bullet)$ is the standard deviation function on the set of weighted average returns, and $R_{s,i}$ is the arithmetic average return through time of the active manager i in the simulation s.

All else being equal, outcome risk parity will tilt a portfolio toward active managers that demonstrate mean reversion in returns, and away from those that show momentum (consistent with constant relative risk aversion utility-style approaches). But if active returns are independent and identically distributed, traditional risk parity and outcome risk parity result in identical portfolios.

When "Greeks" matter: Mean-variance frontier and special cases

In this section, we cover widely used but nontrivial-to-implement methods that evaluate active portfolio construction using mean-variance optimization or the efficient frontier. The latter is a concept in modern portfolio theory that was introduced by Markowitz (1952) and can also be adapted to active portfolio construction.[4] It is a representation of a set of optimal portfolios that offer the maximum expected return (or excess return) for a given level of risk (or excess risk, also known as tracking error). Below, this relationship is captured formulaically:

$$\max_{\{x\}} \left(\mu_P - \gamma \sigma_P^2 \right)$$

In this equation, μ_P is the active managers' expected portfolio return, σ_P^2 is the portfolio variance, and γ is the coefficient of risk aversion.

For this approach, an investor has more inputs to estimate compared with other approaches we discussed earlier, including the investor's level of risk aversion.[5] Practically, an investor would keep changing the level of risk aversion until a certain total active risk threshold is reached.[6]

4 See Garvey, Kahn, and Savi (2017); and Ang, Madhavan, and Ribando (2021).

5 See Liu and Xu (2010) for further insight on how the level of risk aversion can impact optimization, and how it can be tuned and estimated in real-world applications.

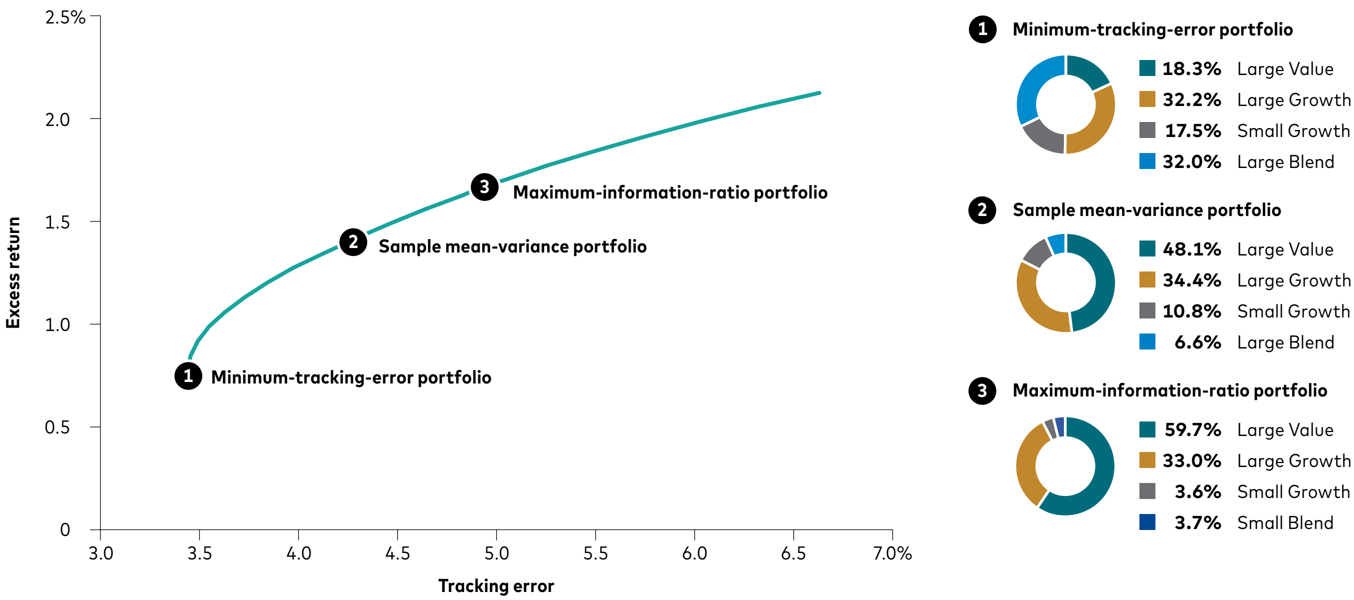
6 A mathematically different but conceptually equivalent approach that would lead to the same solution is minimizing the active portfolio's tracking error subject to a specific level of active outperformance, as reported by Roll (1992).

The efficient frontier is a dynamic concept that not only captures optimal risk/return portfolios but also other portfolio concepts such as minimum variance (or tracking error) and maximum Sharpe (or information) ratio. The efficient frontier can be represented using different sets of risk and return metrics, depending on the goal of the investor. Traditionally, the efficient frontier is depicted as a tradeoff between total risk and total return. But in the context of active portfolio management, it is typically represented using active risk and active return. **Figure 3** depicts an efficient frontier that represents portfolios from an active perspective. This is often referred to as the "tracking-error efficient frontier."

Minimum tracking error and maximum information ratio are on the efficient frontier, and represent two special cases that we want to highlight as potential standalone solutions that do not require a risk-aversion parameter to identify allocation.

FIGURE 3

Depicting the tracking-error (mean-variance) frontier with several portfolios



Notes: Calculations are based on MSCI Barra and December 2023 Vanguard Capital Markets Model (VCMM) 10-year steady-state forecasts. Factor and excess returns are calculated based on historical five-year monthly returns. The VCMM's long-term factor and beta returns were used to estimate total return for individual funds. MSCI Barra was used to estimate tracking error. The sample mean-variance portfolio is an illustrative portfolio on the frontier corresponding to the investor's risk aversion.

Sources: Vanguard calculations, based on data from MSCI Barra.

IMPORTANT: The projections and other information generated by the VCMM regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Distribution of return outcomes from the VCMM are derived from 10,000 simulations for each modeled asset class. Simulations are as of December 2023. Results from the model may vary with each use and over time. For more information, please see the Appendix.

An investor might be interested in investing in active strategies but deviating as little as possible from the passive benchmark. In this case, finding the portfolio that minimizes the tracking error might be the best option. The tracking error is a measure of the risk in an active strategy's portfolio deriving from the investment decisions made by the portfolio manager, and indicates how closely the manager follows the index to which it is benchmarked from a risk perspective. This measure is normally computed as the standard deviation of the difference in returns between the active portfolio and its market benchmark, as detailed in the following formula:

$$TE_P = \sqrt{Var\left[\left(\sum_{i=1}^{n} x_i R_i\right) - R_M\right]}$$

In this equation, R_i is the return of active manager i, and R_M is the return of the market portfolio.

The minimum-tracking-error portfolio lies on the far left of the efficient frontier and represents the portfolio with the lowest active risk on the frontier. This approach requires the estimation of the active funds' risk parameters, which would ultimately mean that an investor has to estimate the beta, factor loadings, specific risk, etc. On the other hand, one of the key advantages of this technique is also its drawback: Excluding expected return or alpha from estimation makes it easier to build the portfolio, but solely focusing on active risk minimization relative to the benchmark could result in overlooking high-risk managers with potentially high alpha. This is because such managers could contribute disproportionally to tracking error.

Another special-case solution that lies on the efficient frontier is the maximum-information-ratio portfolio. Maximizing the portfolio information ratio is another approach very popular among asset allocation practitioners (Grinold, 1989; Kopman and Liu, 2009; Zhang, 2020; and Ang, Madhavan, and Ribando, 2021).

The idea is to weight active managers such that an investor maximizes the ratio of active return to active risk (IR_P). Below is the formulaic representation of this solution:

$$\max_{\{x\}} \frac{\sum_{i=1}^{n}(x_i R_i) - R_M}{TE_P}$$

This approach can sometimes be more challenging from a technical standpoint because it is a nonlinear optimization. One of the main advantages of this technique is its ability to provide a more nuanced assessment of portfolio allocation by considering both returns and risk factors, as well as the relationship between them. On the other hand, even small uncertainty in inputs can lead to statistically different results that are hard to explain.

The mean-variance optimization frontier has served as a cornerstone in portfolio theory for many decades, aiding practitioners in constructing investment portfolios. However, its efficacy can be greatly compromised by data sensitivity, potentially transforming this technique into an "error maximizer" during the asset allocation process. Investors should exercise caution and be confident in their inputs in order to overcome this potentially significant weakness.

Although the objective of this paper is to provide a variety of systematic options to create an active bundle, we would be remiss to not offer at least general guidance on how to choose the "right" strategy when combining active managers. A simple framework for doing so is based on understanding the interplay among investment goals, available data or inputs, and confidence in such data. Investors are encouraged to start with clear strategic goals for the active bundle, and then evaluate data inputs and confidence in those inputs. If confidence is lacking, we would suggest revisiting goals and choosing approaches that rely less heavily on user inputs, since low-quality data can lead to maximizing errors and poor active allocation.

Bringing it all together: Bundling decision to multiasset portfolio allocation

Optimal active bundling

Using the active U.S. equity funds specified in Figure 1 as well as the techniques previously discussed, we have created **Figure 4**, which captures active-bundle allocations and summary statistics for all the bundles.

A notable observation from the active-bundle creation process is the distinct way that bundles are constructed. Some are formed through a lens focusing on total risk and allocation, while others are developed in relationship to a benchmark. It is not unexpected to find that many strategies, which were evaluated from the total portfolio perspective, exhibit a similar allocation. This uniformity can be attributed to the funds that we selected for inclusion in active bundles.

In Figure 4, inverse volatility, absolute risk parity, and outcome risk parity all resulted in very similar allocations. This confirms that when total volatility is the primary risk measure and correlation among managers is high, these approaches tend to converge in their portfolio weights. Once we shift toward techniques that require additional assumptions, such as expected returns or alpha, active-bundle allocation can vary drastically. This highlights the importance of being confident in the assumptions that we make during the bundling process.

FIGURE 4

Assessing active-bundle allocations and summary statistics

	Input-free (1/N) approach	Market capitalization	Inverse volatility	Absolute risk parity	Outcome risk parity	Minimum tracking error	Sample mean-variance	Maximum information ratio
	Total	Total	Total	Total	Total	Relative	Relative	Relative
Bundle weights								
Large Value	20.0%	12.7%	23.6%	23.8%	26.0%	18.3%	48.1%	59.7%
Large Growth	20.0%	40.0%	22.5%	22.4%	23.0%	32.2%	34.4%	33.0%
Small Growth	20.0%	7.3%	16.2%	16.0%	15.0%	17.5%	10.8%	3.6%
Large Blend	20.0%	40.0%	21.5%	21.6%	21.0%	32.0%	6.6%	0.0%
Small Blend	20.0%	0.0%	16.2%	16.2%	15.0%	0.0%	0.0%	3.7%
Statistics								
Total return	9.16%	9.00%	9.22%	9.22%	9.26%	9.05%	9.69%	9.95%
Excess return	0.88%	0.72%	0.94%	0.94%	0.98%	0.77%	1.41%	1.67%
Total volatility	17.81%	16.17%	17.31%	17.31%	17.15%	16.61%	15.86%	15.62%
Excess volatility	4.31%	3.58%	4.00%	4.00%	3.95%	3.44%	4.28%	4.93%
Information ratio	0.20	0.20	0.24	0.24	0.25	0.22	0.33	0.34

Notes: "Total" and "relative" in the column headers refer to whether the optimization used total returns and risk or excess relative to a benchmark. Vanguard calculations are based on MSCI Barra and December 2023 VCMM 10-year steady-state forecasts. Factor and excess returns are calculated based on historical five-year monthly returns. The VCMM's long-term factor and beta returns were used to estimate total return for individual funds. MSCI Barra was used to estimate tracking error. Because different active-bundle approaches have differing objectives, sets of inputs, and assumptions, they cannot be directly compared in terms of in-sample portfolio analytics.

Sources: Vanguard calculations, based on data from MSCI Barra.

Combining an active bundle with passive equity using the VAAM

After an investor has established the allocation among active managers, the next phase involves the estimation of a suitable level of active composition for the overall portfolio. This entails determining the optimal blend between passive equity and the active bundle. For our analysis, we leverage the VAAM, which accounts for active return and an investor's active risk preference.[7] The outcome of the sizing exercise is captured in **Figure 5**, which shows three levels of active risk tolerance: low, medium, and high.

FIGURE 5

Comparing active allocation for different bundles within U.S. equity

	Active risk tolerance		
	Low	**Medium**	**High**
Input-free (1/N) approach	21%	42%	62%
Market capitalization	25%	49%	72%
Inverse volatility	26%	51%	75%
Minimum tracking error	28%	56%	83%
Absolute risk parity	26%	51%	75%
Outcome risk parity	27%	54%	80%
Sample mean-variance	33%	65%	96%
Maximum information ratio	30%	58%	87%

Notes: Calculations are based on MSCI Barra, VAAM, and December 2023 VCMM 10-year steady-state forecasts for U.S. equities to determine the active allocations. Factor and excess returns are calculated based on the historical five-year monthly returns. The VCMM's long-term factor and beta returns were used to estimate total return for individual funds. MSCI Barra was used to estimate excess volatility, also known as tracking error.

Sources: Vanguard calculations, based on data from MSCI Barra.

7 See Aliaga-Díaz et al. (2020). The VAAM is a proprietary model for determining asset allocation—how investments are divided among different assets, such as stocks, bonds, and cash—among active, passive, and factor investment vehicles.

Figure 5 illustrates a distinct relationship between the allocation to active and the level of active preference. As risk tolerance increases, so does the allocation to the active bundle. Another notable observation is the level of active allocation within each category of risk tolerance. The size of the allocation is influenced by critical input variables into the VAAM framework, such as active factor exposures, tracking error, and excess return. In this example, the difference in active allocation is as large as 34% between the input-free approach and the mean-variance bundles. This is not very surprising, given that the profile of these active bundles is significantly different.

We believe that all of the above techniques for bundling active managers are acceptable, and we refrain from endorsing a specific method in this paper. We assert that an active manager bundling decision should be closely aligned with the investor's objectives and the level of confidence in the inputs. At the same time, we emphasize another important aspect: The bundling decision directly influences the extent to which the active bundle will contribute to the allocation in the total portfolio. The primary focus is to pursue a portfolio construction approach that is comprehensive, holistic, and feasible, while accounting for the investor's goals.

Conclusion

This paper presents a comprehensive summary of systematic approaches for combining active managers into active bundles. There is no right or wrong approach, as each method is guided by unique objectives and required inputs. While the methods detailed in this paper are not exhaustive, we have captured a wide spectrum of strategies that cater to diverse investment objectives and preferences. This diversity in strategies provides a rich toolkit for investors and enables them to craft bespoke solutions.

Implementing the methodologies outlined can empower investors to transition from reliance on rule-of-thumb approaches to more systematic and structured methods. The adoption of these methods establishes a framework that demands robust inputs, and requires investors to conduct a thorough examination of their assumptions and potential biases. Adopting a systematic approach that aligns with investment goals encourages a more informed decision-making process, while fostering deeper understanding of the complexities of combining active managers.

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Appendix

FIGURE 6
Pros and cons of optimization techniques

Optimization technique	Pros	Cons
Input-free (1/N) approach	Easy to implement and explain, while offering some level of diversification.	Ignores all return, volatility, and correlation characteristics.
Market capitalization	Simple to construct, cost effective, and remains relatively stable over time, with the ability to move within the style box without materially impacting the overall allocation.	Could lead to high concentration and reduced portfolio diversification for funds with a broad base of exposures. The result may also yield unintended factor exposures that aren't represented by the style box.
Inverse volatility	Takes into account active managers' risk, with the goal of achieving a portfolio with more diversified and balanced risk exposures.	Ignores the relationship among active managers, which might lead to less balance and will overemphasize managers with lower volatility, while overlooking other aspects such as the potential for higher alpha generation.
Minimum tracking error	Aligns expectations to a given benchmark, while accounting for volatility and correlation.	Does not take into account return expectations and may understate allocation to high-alpha managers due to the objective of minimizing active risk (factor and specific).
Risk parity	Provides increased diversification that allows each asset to contribute equally to the total risk of the portfolio.	May lead to lower returns, given the priority of managing risk versus maximizing returns.
Outcome risk parity	Provides increased diversification that allows each asset to contribute equally to the total risk of the portfolio over a multiperiod horizon. Contrary to risk parity, it allows for return predictability.	May lead to lower returns, given the priority of managing risk versus maximizing returns.
Sample mean-variance	Accounts for all key statistics: return, volatility, and correlation.	Requires more inputs, which can lead to error maximization and poor realized results.
Maximum information or Sharpe ratio	Accounts for all key statistics: return, volatility, and correlation.	Requires more inputs, which can lead to error maximization and poor realized results.

Source: Vanguard.

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